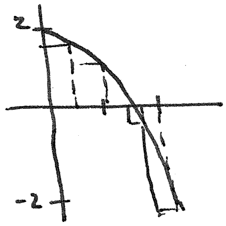
hw set 3
sect 5.2
p 364

1. $f(x) = 2 - x^2$ , $0 \le x \le 2$
$n = 4$ and right endpoints




$R_4 = \Delta x\left(f(\tfrac{1}{2}) + f(1) + f(\tfrac{3}{2}) + f(2)\right)$
$= \tfrac{1}{2}(1.75 + 1 + -.25 + -2)$
$= .25$

4. $f(x) = x - 2\sin 2x$ , $[0,3]$ , $n = 6$
round to 6 decimal places

a) $R_6 = \Delta x\left(f(\tfrac{1}{2}) + f(1) + f(3/2) + f(2) + f(5/2) + f(3)\right)$
$= \tfrac{1}{2}(10.7065077)$
$= 5.353253848 = 5.353254$

b) $M_6 = \Delta x\left[f(\tfrac{1}{4}) + f(3/4) + f(1.25) + f(1.75) + f(2.25) + f(2.75)\right]$
$= \tfrac{1}{2}[8.916922003]$
$= 4.458461002 = 4.458461$

12. $\int_1^5 (\underbrace{x^2 e^{-x}}_{=f(x)})\,dx$ , $n = 4$, Midpoint
$\Delta x = \frac{5-1}{4} = 1$
$M_4 = \Delta x\left(f(1\tfrac{1}{2}) + f(2\tfrac{1}{2}) + f(3\tfrac{1}{2}) + f(4\tfrac{1}{2})\right)$
$= 1[1.609949229]$
$= 1.6099$

17. $\lim_{n\to\infty} \sum_{i=1}^{n} x_i \sin x_i \,\Delta x$ , $[0, \pi]$
$= \int_0^{\pi} x \sin x \, dx$

18. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{e^{x_i}}{1 + x_i}\Delta x$, $[1,5]$
$= \int_1^5 \frac{e^x}{1+x}\,dx$

22. $\int_1^4 (x^2 + 2x - 5)\,dx$
$= \lim_{n\to\infty} \sum_{i=1}^{n} (x_i^2 + 2x_i - 5)\left(\frac{4-1}{n}\right)$
$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left[(1 + \tfrac{3i}{n})^2 + 2(1 + \tfrac{3i}{n}) - 5\right] \cdot \tfrac{3}{n}\right]$
$= \lim_{n\to\infty} \left(\sum_{i=1}^{n} \left[\frac{9i^2}{n^2} + \frac{12i}{n} - 2\right] \cdot \frac{3}{n}\right)$
$= \lim_{n\to\infty} \left(\frac{27}{n^3}\sum_{i=1}^{n} i^2 + \frac{36}{n^2}\sum_{i=1}^{n} i - \frac{6}{n}\sum_{i=1}^{n} 1\right)$
$= \lim_{n\to\infty} \left(\frac{27}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{36}{n^2} \cdot \frac{n(n+1)}{2} + \frac{-6}{n} \cdot n\right)$
$= \lim_{n\to\infty} \left(\frac{54n^3 + 81n^2 + 27n}{6n^3} + \frac{36n^2 + 36n}{2n^2} - 6\right)$
$= \frac{54}{6} + 18 - 6 = 21$

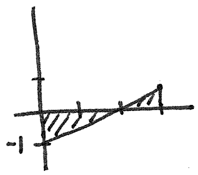
28. $\int_1^{10} (x - 4\ln x)\,dx$
$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$
$\begin{cases} \Delta x = \frac{10-1}{n} = \frac{9}{n} \\ x_i = 1 + i\Delta x = 1 + 9i/n \end{cases}$
$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + \frac{9i}{n} - 4\ln\left(1 + \frac{9i}{n}\right)\right)\frac{9}{n}$

32. a) $\int_0^2 g(x)\,dx$
$= 2 \cdot 4 \cdot \tfrac{1}{2} = 4$

b) $\int_2^6 g(x)\,dx$
$= -\tfrac{1}{2}(\pi \cdot 2^2) = -2\pi$

c) $\int_0^7 g(x)\,dx$
$= 4 + 2\pi + \tfrac{1}{2} = 4.5 + 2\pi$

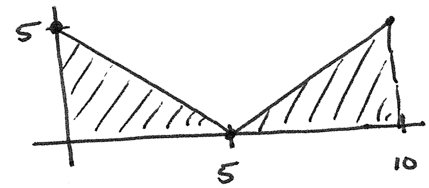
33. $\int_0^3 \left(\frac{1}{2}x - 1\right)dx$




$= -\left(1 \cdot 2 \cdot \frac{1}{2}\right) + \frac{1}{2}\left(1 \cdot \frac{1}{2}\right)$

$= \frac{1}{4} - 1 = -3/4$

38. $\int_0^{10} |x-5|\, dx$




$= \frac{1}{2}(5 \cdot 5) + \frac{1}{2}(5 \cdot 5)$

$= 25$

39. $\int_4^9 \sqrt{x}\, dx = \frac{38}{3}$

$\int_9^4 \sqrt{x}\, dx = -\frac{38}{3}$

40. $\int_1^1 x^2 \cos x\, dx = 0$

41. $\int_{-2}^{2} f(x)dx + \int_2^5 f(x)\, dx - \int_{-2}^{-1} f(x)\, dx$

$= \int_{-2}^{5} f(x)\, dx - \int_{-2}^{-1} f(x)$

$= \int_{-1}^{5} f(x)\, dx$

42. $\int_1^5 f(x)\, dx = 12$, $\int_4^5 f(x)\, dx = 3.6$

$\int_1^4 f(x)\, dx = 12 - 3.6 = 8.4$

43. $\int_0^9 f(x)dx = 37$, $\int_0^9 g(x)\, dx = 16$

$\int_0^9 [2f(x) + 3g(x)]\, dx$

$= 2\int_0^9 f(x)\, dx + 3\int_0^9 g(x)\, dx$

$= 2 \cdot 37 + 3 \cdot 16$

$= 122$